國立中正大學
113學年度碩士班招生考試
試 題
[第1節]

科目名稱	通訊原理
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

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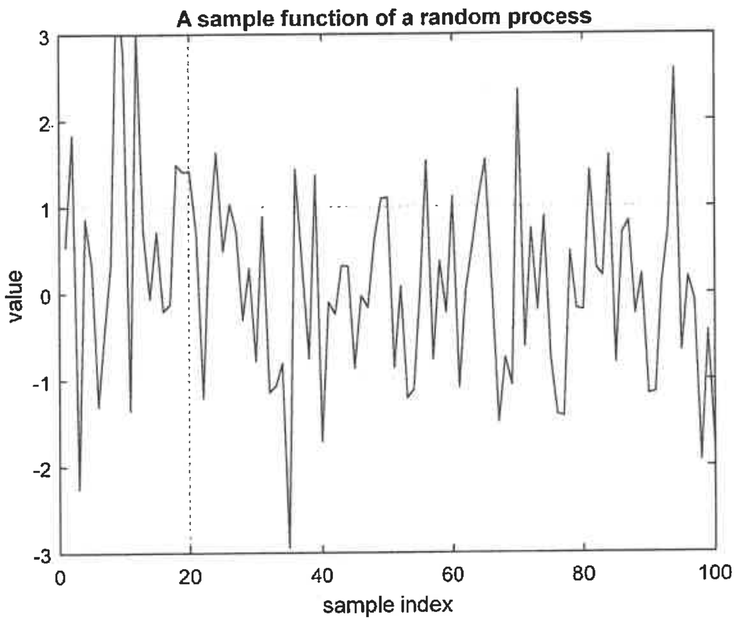
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1. (20 %) The additive white Gaussian noise (AWGN) $n(t)$ appears in communication system.
   (a) (10 %) Please give a mathematical definition for the AWGN $n(t)$.
   (b) (10 %) Suppose now you are given a sample function $n(t),\ t = 0,1,\cdots,N-1$, depicted below. Is it a sample function of an AWGN $n(t)$? Please address the steps and methods that you use to justify your answer.



2. (20 %) Consider a periodic signal $x(t)$ with period T_0. Over one period, the

$$x(t) = \Pi\left(\frac{2t}{T_0}\right) - \frac{1}{2}, \quad -\frac{T_0}{2} < t < \frac{T_0}{2},$$

where $\Pi(\cdot)$ is the unit rectangular pulse. The signal $x(t)$ is filtered by an ideal lowpass filter with cutoff frequency $\frac{3}{2T_0}$ to produce output signal $y(t)$.

   (a) (2 %) Plot $x(t)$ for $T_0 = 1$.
   (b) (5 %) What is the output signal $y(t)$?
   (c) (10 %) What is the autocorrelation of the output signal $y(t)$?
   (d) (3 %) What is the power spectral density of the output signal $y(t)$?

3. (20 %) The frequency modulated signal is denoted by $x(t) = A_c \cos\left(2\pi f_c t + 2\pi f_d \int^t m(\tau)d\tau\right)$, where f_c is the carrier frequency, f_d is the frequency deviation constant, and $m(t)$ is the message signal. Consider now $m(t) = A_m \cos(2\pi f_m t)$.

   (a) (2 %) Show that the modulated signal may be expressed as $x(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$. Please give the value of β.

   (b) (3 %) What is the complex envelope of $x(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$?

   (c) (10 %) What is the continuous-time Fourier transform of $x(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$? You may use Bessel function to express your result.

   (d) (5 %) What is the time average power of $x(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$?

4. (20 %) The received (complex-valued) baseband signal is $x = hs + w$, where h is the known channel gain, w is the additive white Gaussian noise with zero mean and variance σ^2. The transmitted symbol s comes from the 8-PSK constellation set $S = \{e^{j2\pi k/8} : k = 0,1,\cdots,7\}$.

   (a) (2 %) What is the constellation diagram for S?
   (b) (3 %) What is the likelihood function for detecting transmitted s from received x?
   (c) (5 %) What is the decision rule for the maximum likelihood detector (MLD) to detect transmitted s? Please reduce to the simplest form as possible as you can.
   (d) (5 %) What are the decision regions for the MLD?
   (e) (5 %) What is the symbol error probability for the MLD?

5. (20 %) A stereo frequency modulation (FM) broadcasting is to transmit a left channel signal $m_1(t)$, a pilot $p(t) = \cos(2\pi f_p t)$, and a right channel signal $m_2(t)$ at a time through frequency division multiplexing and frequency modulation. Assume that $m_1(t)$ and $m_2(t)$ are of the same bandwidth 15 kHz and pilot frequency is $f_p = 19$ kHz.

   (a) (10 %) Give your design for the stereo FM transmitter using a block diagram. You may give spectra of signals in the block diagram if necessary.
   (b) (10 %) Give your design for the stereo FM receiver using a block diagram. You may give spectra of signals in the block diagram if necessary.

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試 題

[第 2 節]

科目名稱	線性代數
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

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科目名稱：線性代數　　　　本科目共 1 頁　第 1 頁

系所組別：通訊工程學系-通訊甲組

1. Let $C=\begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}$, $D=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $E=\begin{bmatrix} 0 & a \\ 1 & b \end{bmatrix}$. Answer the following questions with the appropriate matrix names (***C***, ***D*** or ***E***). Note: No partial scores are given for each question.
   a. (5 pts.) Identify matrices that are row equivalent when a is 1 and b is 0.
   b. (5 pts.) Determine which matrix has $\{\boldsymbol{0}\}$ as the orthogonal complement of its row space when a is 0 and b is 1.
   c. (5 pts.) Which of these matrices satisfies the condition that the rank plus the nullity equals 2?
   d. (5 pts.) Which matrix is not full-rank when a and b are 1's?
   e. (5 pts.) Identify the matrix that contains row vectors that can span R^2 for all real numbers a and b, given that a is not equal to b.
   f. (5 pts.) Determine which matrix could be singular, given that the product of a and b is not equal to 0.

2. Determinant identity
   a. (10 pts.) For a given matrix $\boldsymbol{A}=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, identify all possible matrices ***B***, where the last row of ***B*** is ***0***, such that the determinants of ***AB*** and ***BA*** are equal.
   b. (25 pts.) Prove that if ***A*** and ***B*** are matrices of sizes $m\times n$ and $n\times m$, then $\det(\boldsymbol{I}_m+\boldsymbol{AB})=\det(\boldsymbol{I}_n+\boldsymbol{BA})$.
   Hint: $\begin{bmatrix} I_n & -B \\ A & I_m \end{bmatrix}$.

3. Let $T_1 : \mathbf{P}_1 \rightarrow \mathbf{P}_2$ be the linear transformation defined by $T_1(\mathbf{p}(x)) = x{\cdot}\mathbf{p}(x)$ and let $T_2 : \mathbf{P}_n \rightarrow \mathbf{P}_n$ be the linear operator defined by $T_2(\mathrm{p}(x)) = \mathrm{p}(x+1)$, where $B = \{1, 2x\}$ and $B' = \{1, x, 2x^2\}$ are bases for $\mathbf{P}_1$ and $\mathbf{P}_2$, respectively. Every case requires detailed information.
   a. (10 pts.) Determine the coordinate vectors of $(x+1)_B$ and $(x^2+x)_{B'}$.
   b. (10 pts.) Represent the linear transformation T_1 from $\mathbf{P}_1$ to $\mathbf{P}_2$ as the matrix $[T_1]_{B\rightarrow B'}$.
   c. (10 pts.) Find the matrix representation $[T_2]_{B'}$ of the linear transformation T_2 in $\mathbf{P}_2$.
   d. (5 pts.) Show the matrix representation of the composition of two linear transformations $[T_2 \circ T_1]_{B\rightarrow B'}$.

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[第2節]

科目名稱	機率
系所組別	通訊工程學系-通訊甲組

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1) **(15 points)** Let $\Omega = \{1, 2, 3, 4, 5, 6\}$ be the sample space. Define three events: $A = \{1, 2\}$, $B = \{2, 3\}$, and $C = \{4, 5, 6\}$. The probability measure is unknown, but it satisfies the three axioms of probability.
   a) **(5 points)** What is the probability of $A \cap C$?
   b) **(5 points)** What is the probability of $A \cup B \cup C$?
   c) **(5 points)** State a condition on the probability of either B or C that would allow them to be independent events.

2) **(20 points)** The probability density function (pdf) of a random variable X is shown in Fig. 1.

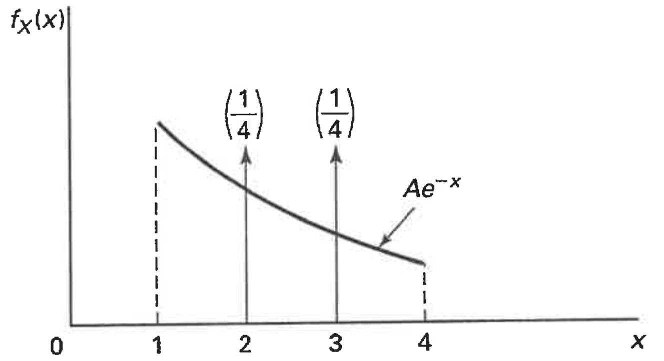

Fig. 1: pdf of X

   a) **(5 points)** Compute the value of A.
   b) **(10 points)** Find the cumulative distribution function (cdf) of X, that is, $F_X(x)$.
   c) **(5 points)** Compute the probability of the event $\{2 \le X < 3\}$.

3) **(25 points)** Consider a communication channel corrupted by noise. Let X be the value of the transmitted signal and Y the value of the received signal. Assume that the conditional density of Y given X is Gaussian, that is,

$$f_{Y|X}(y|x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-x)^2}{2\sigma^2}\right)$$

and that X takes on only the values $+1$ and -1 equally likely.
   a) **(5 points)** Find the pdf of X, that is, $f_X(x)$.
   b) **(10 points)** Find the pdf of Y, that is, $f_Y(y)$.
   c) **(10 points)** What is the conditional density of X given Y, that is, $f_{X|Y}(x|y)$.

4) **(20 points)** Let X_1 and X_2 be independent and exponentially distributed random variables with pdf

$$f_{X_i}(x) = \frac{1}{\mu} \exp\left(-\frac{x}{\mu}\right) u(x), \quad i = 1, 2,$$

where $\mu > 0$ and $u(x)$ is the unit step function. Define $Z \triangleq \max(X_1, X_2)$.

a) **(10 points)** Find the cdf of Z, that is, $F_Z(z)$.

b) **(10 points)** Find the pdf of Z, that is, $f_Z(z)$.

5) **(20 points)** In your physics courses, you have studied the concept of momentum $p = mv$ in the deterministic, that is, nonrandom sense. In reality, measurements of mass m and velocity v are never precise, thereby giving rise to an unavoidable uncertainty in these quantities. In this problem, we treat these quantities as random variables. So, consider a random variable mass M with given pdf $f_M(m)$ and a random variable velocity V with given pdf $f_V(v)$. We are also given the averages $\mu_M = E[M]$ and $\mu_V = E[V]$ (that would presumably correspond to our measurements in the physics course). Assume that M and V are independent and nonnegative random variables.

a) **(5 points)** Express the cdf of the momentum $P = MV$, that is, $F_P(p)$, in terms of the known pdf's $f_M(m)$ and $f_V(v)$.

b) **(10 points)** Find the pdf of $P = MV$, that is, $f_P(p)$, in terms of $f_M(m)$ and $f_V(v)$.

c) **(5 points)** Determine the expected value of the momentum $\mu_P = E[P]$ as a function of μ_M and μ_V.