國立中正大學
110 學年度碩士班招生考試
試 題

[第 2 節]

科目名稱	線性代數與計算機組織
系所組別	通訊工程學系-通訊乙組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

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5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

Linear Algebra

1. Consider an overdetermined system

$$x + 2y = 1$$
$$2x + y = a$$
$$x + y = b$$
$$x + 3y = c$$

In **R**, find all possible a, b, and c for two cases,

a. (10 pts.) consistent and

b. (5 pts.) inconsistent.

2. Let $\mathbf{S} = \{(x_1, x_2, x_3, x_4) \in \mathbf{R}^4 \mid x_2 + x_3 + x_4 = 0\}$ and $\mathbf{T} = \{(x_1, x_2, x_3, x_4) \in \mathbf{R}^4 \mid 2x_1 = x_2, x_3 + x_4 = 0\}$. Find the basis and dimension of each subspace.

a. (5 pts.) **S**.

b. (5 pts.) **T**.

c. (5 pts.) **S** $\cup$ **T**.

d. (5 pts.) **S** $\cap$ **T**.

3. Suppose that $\det \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{bmatrix} = 24$.

a. (10 pts.) Find $a_i \in \mathbf{R}$, $i = 1, 2, 3$, and 4.

b. (5 pts.) Find the determinant of $\begin{bmatrix} 2a_1 & 2a_2 & 2a_3 & 2a_4 \\ 0 & -1 & 0 & 3 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

Computer Organization

4. A memory management unit (MMU) is a computer hardware unit having all memory references passed through itself, primarily performing the translation of virtual memory addresses to physical addresses.
a. (5 pts.) Describe virtual memory addresses and physical addresses. What is the relationship between virtual memory addresses and physical addresses and how to translate them?
b. (5 pts.) Describe Translation Lookaside Buffer (TLB). What role does it play in the translation of virtual memory addresses to physical addresses?
c. (5 pts.) What are similarities and differences between CPU Cache and TLB?

5. (15 pts.) Bus Arbitration refers to the process by which the current bus master accesses and then leaves the control of the bus and passes it to the another bus requesting processor unit. A conflict may arise if the number of DMA controllers or other controllers or processors try to access the common bus at the same time, but access can be given to only one of those. Only one processor or controller can be the bus master at the same point of time. Describe and design a circuit of bus arbitration for four processor units, each unit with a bus request pin and a bus grant pin. (Hint: daisy-chain and parallel arbitration logic are two possible solutions.)

6. Consider the following two processors. P1 has a clock rate of 4 GHz, average CPI of 0.9, and requires the execution of 5.0E9 instructions. P2 has a clock rate of 3 GHz, an average CPI of 0.75, and requires the execution of 1.0E9 instructions.
a. (5 pts.) One usual fallacy is to consider the computer with the largest clock rate as having the largest performance. Check if this is true for P1 and P2.
b. (5 pts.) Another fallacy is to consider that the processor executing the largest number of instructions will need a larger CPU time. Considering that processor P1 is executing a sequence of 1.0E9 instructions and that the CPI of processors P1 and P2 do not change, determine the number of instructions that P2 can execute in the same time that P1 needs to execute 1.0E9 instructions.
c. (5 pts.) A common fallacy is to use MIPS (millions of instructions per second) to compare the performance of two different processors, and consider that the processor with the largest MIPS has the largest performance. Check if this is true for P1 and P2.
d. (5 pts.) Another common performance figure is MFLOPS (millions of floating-point operations per second), defined as

MFLOPS = No. FP operations / (execution time x 1.0E6) .

Assume that 50% of the instructions executed on both P1 and P2 are floating-point instructions. Find the MFLOPS figures for the programs.

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[第 2 節]

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系所組別	通訊工程學系-通訊乙組

－作答注意事項－

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國立中正大學 110 學年度碩士班招生考試試題

科目名稱：線性代數與資料結構.　　本科目共 1 頁　第 1 頁

系所組別：通訊工程學系-通訊乙組

Linear Algebra

1. Consider an overdetermined system

$$x + 2y = 1$$
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$$x + 3y = c$$

In **R**, find all possible a, b, and c for two cases,

a. (10 pts.) consistent and

b. (5 pts.) inconsistent.

2. Let $\mathbf{S} = \{(x_1, x_2, x_3, x_4) \in \mathbf{R}^4 \mid x_2 + x_3 + x_4 = 0\}$ and $\mathbf{T} = \{(x_1, x_2, x_3, x_4) \in \mathbf{R}^4 \mid 2x_1 = x_2, x_3 + x_4 = 0\}$. Find the basis and dimension of each subspace.

a. (5 pts.) **S**.

b. (5 pts.) **T**.

c. (5 pts.) $\mathbf{S} \cup \mathbf{T}$.

d. (5 pts.) $\mathbf{S} \cap \mathbf{T}$.

3. Suppose that $\det\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 3 \end{bmatrix} = 24$.

a. (10 pts.) Find $a_i \in \mathbf{R}$, $i = 1, 2, 3$, and 4.

b. (5 pts.) Find the determinant of $\begin{bmatrix} 2a_1 & 2a_2 & 2a_3 & 2a_4 \\ 0 & -1 & 0 & 3 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

Data Structure

4. An undirected graph with n vertices is represented by an n by n adjacency matrix G. If there is an arc between vertex i and vertex j, the entries $G[i][j]$=1 and $G[j][i]$=1. Otherwise, $G[i][j]$=0 and $G[j][i]$=0.

a. (15 pts.) Using C or pseudocode to implement the sub-routine *check_path*(G, n, x, y), which checks whether there is a path from vertex x to vertex y in the given graph G.

b. (15 pts.) Using C or pseudocode to implement the sub-routine *k_hops_neighbors*(G, n, x, k), which displays all the vertices that vertex x can reach within k hops.

5. X is an integer array with n different elements ($X[0],\ldots,X[n-1]$). The elements in X are arranged in descending order, so if $i<j$ then $X[i]>X[j]$.

a. (15 pts.) Using C or pseudocode to implement the sub-routine *BS*(X, n, k) that uses binary search algorithm to find the index of the given integer k. *BS* returns the index of the element whose value is k. If X does not contain k, the program returns -1.

b. (5 pts.) What is the time complexity of *BS*.

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[第2節]

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系所組別	通訊工程學系-通訊乙組

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I. 選擇題：(共50分)單選題，每答對一題得五分

Answer the questions below by providing the most appropriate choice. Write down the correct answer on your answer sheet. No explanations will be considered in grading this portion of the exam. Each correct answer is worth 5 points.

1. An analog source output may be converted into a digital form and the message can be transmitted via digital modulation and demodulated as a digital signal at the receiver. Which one is NOT the advantage to transmitting an analog signal by means of digital modulation?

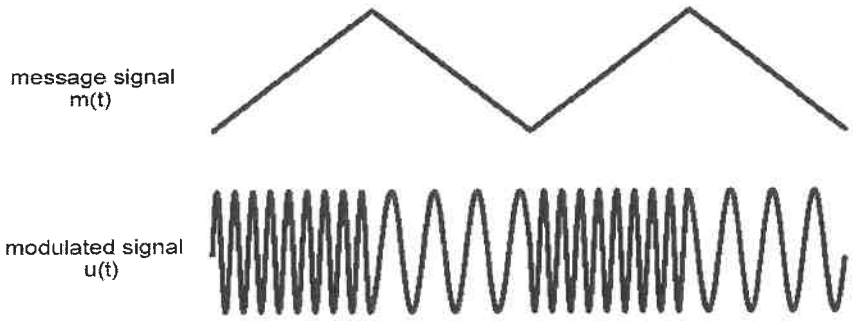
   (a) Eliminates effects of noise at each regeneration point.

   (b) Signal fidelity is better controlled.

   (c) Redundancy may be removed prior to modulation, which saves the channel bandwidth.

   (d) The receiver does not need to reconstruct the analog signal. Therefore, the complexity of the receiver can be reduced.

2. Let $m(t)$ be the message signal and $u(t)$ be the modulated signal. The signals $m(t)$ and $u(t)$ are illustrated in the following figure. What is the modulation scheme used in this modulator?



   (a) Frequency modulation

   (b) Phase modulation

   (c) Amplitude modulation

   (d) Single-sideband modulation

   (e) None of the above.

3. To prevent the effect of aliasing in practice, which of the following method is correct?

   (a) Use a high-pass filter to attenuate the in-band noise.

   (b) Use a low-pass filter to attenuate those high frequency components of the signal before sampling.

   (c) The signal is sampled at a rate slightly lower than the Nyquist rate.

   (d) Reduce the signal amplitude so that aliasing effect can be reduced.

   (e) None of the above.

4. A quadrature-carrier multiplexing allows us to transmit two message signals $m_1(t)$ and $m_2(t)$ on the same frequency. The transmitted signal can be expressed as

$$u(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t).$$

   Describe how to demodulate $m_1(t)$ from $u(t)$.

   (a) Use an envelope detector followed by a low-pass filter.

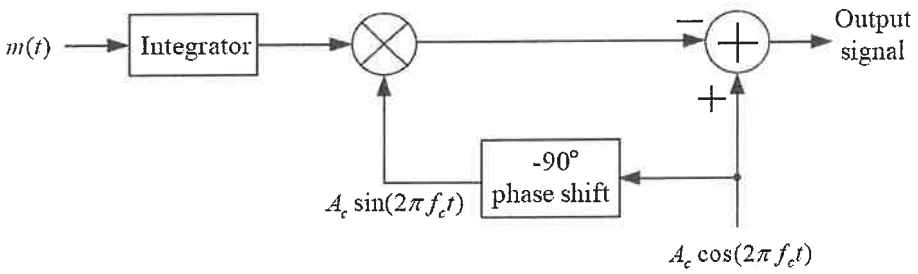
   (b) Multiply the received signal by $\sin(2\pi f_c t)$ and then pass the signal through a low-pass filter.

   (c) Multiply the received signal by $\cos(2\pi f_c t)$ and then pass the signal through a low-pass filter.

   (d) Multiply the received signal by $\cos(2\pi f_c t)$ and then pass the signal through an envelope detector.

   (e) None of the above.

5. Let $m(t)$ be the message signal. What is the function of the following block diagram?



   (a) It is a sigma-delta modulator.

   (b) It is a narrow band FM modulator.

   (c) It is a differential PCM transmitter.

   (d) It is a narrow band PM modulator.

   (e) None of the above.

6. Consider two digital communication systems, the M-FSK and M-PSK, under the additive white Gaussian noise (AWGN) channel. Which of the following statement is true?

   (a) For M-FSK under the same E_b/N_0, the symbol error rate decreases as M increases.

   (b) For M-PSK under the same E_b/N_0, the symbol error rate decreases as M increases.

   (c) For M-FSK under the same data rate, the bandwidth decreases as M increases.

   (d) For M-PSK under the same data rate, the bandwidth increases as M increases.

   (e) None of the above.

7. Assume M signals $s_1(t), s_2(t), \ldots, s_M(t)$ are employed in a digital communication system over an AWGN channel with noise power-spectral density $N_0/2$. Let P_m be the a priori probability of transmitting $s_m(t)$ and $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ be the orthonormal basis for this signal set. The equivalent vector form channel is represented as

$$\mathbf{r} = \mathbf{s}_m + \mathbf{n}$$

   Which of the following statement is untrue?

   (a) The maximum-likelihood (ML) detection rule is

$$\hat{m} = \arg \min_{1 \le m \le M} \|\mathbf{r} - \mathbf{s}_m\|^2$$

   (b) The maximum a posteriori probability (MAP) detection rule is

$$\hat{m} = \arg \max_{1 \le m \le M} \left(N_0 \ln P_m - \|\mathbf{r} - \mathbf{s}_m\|^2 \right)$$

   (c) The MAP detection rule and the ML detection rule are equivalent if $P_m = 1/M$, for $m = 1, \ldots, M$.

   (d) If all signals, $s_1(t), s_2(t), \ldots, s_M(t)$, have the same energy, the ML detection rule can be represented as

$$\hat{m} = \arg \min_{1 \le m \le M} \mathbf{r} \cdot \mathbf{s}_m$$

8. Let $s(t)$ be a real-valued transmitted signal. The observed signal can be represented as

$$r(t) = s(t) + z(t)$$

   where $z(t)$ is a white Gaussian noise with power spectral density $N_0/2$. The received signal is passed through a filter that has an impulse response $h(t)$ which results the output signal $y(t) = \int_{-\infty}^{\infty} r(\tau) h(t - \tau) d\tau$. Which of the following statement is true?

   (a) If $h(t) = s(t)$, the sampled signal $y(T)$ has the maximum signal-to-noise ratio.

   (b) If $h(t) = s(T - t)$, the sampled signal $y(T)$ has the maximum signal-to-noise ratio.

   (c) If $h(t) = s(t - T)$, the sampled signal $y(T)$ has the maximum signal-to-noise ratio.

   (d) If $h(t) = s(-t)$, the sampled signal $y(T)$ has the maximum signal-to-noise ratio.

   (e) None of the above.

9. In a PAM system, the signal waveforms can be represented as

$$s_m(t) = A_m p(t), \quad 1 \le m \le M$$

where $p(t)$ is a real-valued pulse of duration T and $A_m, 1 \le m \le M$ denotes the set of M possible amplitudes corresponding to $M = 2^k$ possible k-bit blocks of symbols. The signal amplitudes A_m take the discrete values $A_m = 2m - 1 - M$, for $m = 1, \ldots, M$. Define $E_p = \int_0^T p^2(t)dt$. Which of the following statement is untrue?

(a) PAM signals are one-dimensional with $\phi(t) = p(t)/\sqrt{E_p}$ as the basis.

(b) Define $E_{\text{avg}} = \frac{E_p}{M} \sum_{m=1}^{M} A_m^2$. Then $E_{\text{avg}} = \frac{(M^2-1)E_p}{3}$.

(c) Define $d_{mn}^2 = \int_0^T |s_m(t) - s_n(t)|^2 dt$. Then $d_{mn}^2 = E_p |A_m - A_n|$.

(d) Under the same signal-to-noise ratio, the symbol error rate increases as M increases.

10. Assume M signals $s_1(t), s_2(t), \ldots, s_M(t)$ are employed for digital communication over an AWGN channel with noise power-spectral density $N_0/2$. The M signals are represented by

$$s_m(t) = \sqrt{E_s} \cos\left(\frac{2\pi(m-1)}{M}\right) \phi_1(t) + \sqrt{E_s} \sin\left(\frac{2\pi(m-1)}{M}\right) \phi_2(t)$$

for $0 \le t \le T$, where $\phi_1(t) = \sqrt{2/T}\cos(2\pi f_c t)$ and $\phi_2(t) = -\sqrt{2/T}\sin(2\pi f_c t)$ and $f_c = n/T$ with n as a positive integer. Which of the following statement is untrue?

(a) The signal $s_m(t)$ can also be represented as

$$s_m(t) = \sqrt{2E_s/T} \cos\left(2\pi f_c t - \frac{2\pi(m-1)}{M}\right)$$

(b) $\phi_1(t)$ and $\phi_2(t)$ are orthogonal.

(c) In vector representation,

$$\mathbf{s}_m = \left[\sqrt{E_s}\cos(2\pi(m-1)/M), \sqrt{E_s}\sin(2\pi(m-1)/M)\right]$$

(d) Define $d_{mn}^2 = \int_0^T |s_m(t) - s_n(t)|^2 dt$. Then $d_{mn}^2 = 2E_s\left(1 - \cos\left(\frac{2\pi(m-n)}{M}\right)\right)$.

II. 計算題（共50分）

Give detailed derivations on the following questions. The grade of this portion depends not only on the correct answers but also on the explanations and derivations. Therefore, explain every detail as possible as you can.

1. (25 %) A QPSK communication system over an AWGN channel uses one of the four equiprobable signals $s_M(t) = A\cos(2\pi f_c t + \pi(m-1)/2)$, where $m = 1, 2, 3, 4$, f_c is the carrier frequency, and the duration of each signal is T. The power spectral density of the channel noise is $N_0/2$.

   (a) (10 %) What is the symbol error probability of this system in terms of A, T, and N_0?

   (b) (10 %) If Gray coding is used, what is the bit error probability in terms of A, T, and N_0?

   (c) (5 %) What is the theoretical minimum required transmission bandwidth for this communication system?

2. (25 %) The received signal in a binary communication system that employs antipodal signals of duration T is

$$r(t) = Ag(t) + n(t), \quad \text{for } 0 \le t < T$$

   where $n(t)$ is the additive white Gaussian noise with power spectral density $N_0/2$, and

$$g(t) = \begin{cases} \sqrt{2/T} & T/2 \le t < T \\ 0 & \text{otherwise} \end{cases}$$

   The value of A is given by

$$A = \begin{cases} +\alpha & \text{if 1 is transmitted} \\ -\alpha & \text{if 0 is transmitted} \end{cases}$$

   (a) (5 %) Sketch the impulse response of the filter matched to $g(t)$.

   (b) (5 %) Sketch the output of the matched filter to the input $g(t)$.

   (c) (5 %) Determine the variance of the noise output of the matched filter at $t = T$ with $r(t)$ as the input.

   (d) (5 %) Let $y(t)$ be the output of the matched filter with $r(t)$ as the input. Assume that symbol 1 occurs with probability p. Determine the maximum a posteriori probability (MAP) detection rule given the sampled signal $y(T)$.

   (e) (5 %) Determine the bit error probability for the MAP detection rule in terms of p, T, α, and N_0.

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試 題

[第3節]

科目名稱	通訊原理
系所組別	通訊工程學系-通訊甲組

－作答注意事項－

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科目名稱：通訊原理　　本科目共 4 頁　第 1 頁

系所組別：通訊工程學系-通訊甲組

I. 選擇題：(共50分)單選題，每答對一題得五分

Answer the questions below by providing the most appropriate choice. Write down the correct answer on your answer sheet. No explanations will be considered in grading this portion of the exam. Each correct answer is worth 5 points.

1. Let the power spectral density of $m(t)$ be $S_m(f)$. Determine the power spectral density of the signal $u(t) = A_c m(t)\cos(2\pi f_c t)$.

   (a) $\frac{A_c^2}{2}[S_m(f-f_c) - S_m(f+f_c)]$

   (b) $\frac{A_c^2}{4}[S_m(f-f_c) + S_m(f+f_c)]$

   (c) $A_c^2[S_m(f-f_c) + S_m(f+f_c)]$

   (d) $\frac{A_c}{4}[S_m(f-f_c) + S_m(f+f_c)]$

   (e) None of the above.

2. Let $m(t)$ be the message signal and $\hat{m}(t)$ be the Hilbert transform of $m(t)$. An upper single-sideband amplitude-modulated (USSB-AM) signal can be expressed as

   (a) $A_c m(t)\cos(2\pi f_c t) - A_c \hat{m}(t)\sin(2\pi f_c t)$

   (b) $A_c m(t)\cos(2\pi f_c t) + A_c \hat{m}(t)\sin(2\pi f_c t)$

   (c) $A_c m(t)\cos(2\pi f_c t) + A_c \hat{m}(t)\cos(2\pi f_c t)$

   (d) $A_c m(t)\sin(2\pi f_c t) + A_c \hat{m}(t)\sin(2\pi f_c t)$

   (e) None of the above.

3. Let $m(t)$ be the message signal and P_m be the power of $m(t)$. The FM signal can be represented as $u(t) = A_c\cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right)$. The power of the transmitted signal is

   (a) $(1/2)A_c^2$

   (b) $2A_c^2 P_m$

   (c) $A_c^2 P_m/2$

   (d) $(1/4)A_c^2$

   (e) None of the above.

4. The power spectral density of a white noise $N_w(t)$ is $N_0/2$. Let $N_w(t)$ be the input of a filter with frequency response

$$H(f) = \begin{cases} A, & |f - f_c| \le W \\ 0. & \text{otherwise} \end{cases}$$

   where $f_c > W$. The output of this filter is denoted as $N(t)$. Determine the power of $N(t)$.

   (a) $A^2 W N_0 f_c$

   (b) $2WN_0A^2$

   (c) $WN_0/(2f_c)$

(d) WN_0A^2

(e) None of the above.

5. Regarding to the anti-aliasing filter, which of the following statements is true?

(a) The anti-aliasing filter is a high pass filter to remove the in-band noise.

(b) The anti-aliasing filter is a combination of non-aliasing high-pass and low-pass filters.

(c) The anti-aliasing filter is a low-pass filter to prevent aliasing effect after sampling.

(d) The anti-aliasing filter is a combination of two non-aliasing high-pass filters.

(e) None of the above.

6. Let $s_m(t) = A\cos(2\pi f_c t + 2\pi(m-1)/M)$ for $m = 1, \ldots, M$ and $0 \le t < T$. Which of the following statement is true.

(a) All signals $s_m(t)$, for $m = 1, \ldots, M$, have the same energy

$$E_m = \int_0^T |s_m(t)|^2 dt = A^2T$$

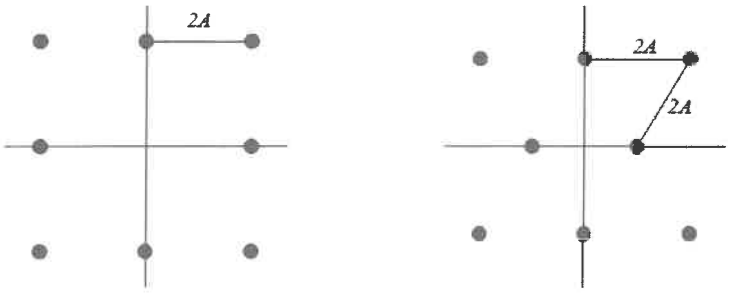
(b) All signals $s_m(t)$, for $m = 1, \ldots, M$ can be represented as linear combinations of one basis function for $M > 2$.

(c) $\int_0^T |s_m(t) - s_n(t)|^2 dt = \sqrt{A^2T\left(1 - \cos\left(\frac{2\pi(m-n)}{M}\right)\right)}$.

(d) The minimum Euclidean distance of this signal set is $\sqrt{2A^2T/M}$.

(e) None of the above.

7. Consider two 8-point QAM signal constellations shown below. The minimum Euclidean distance between adjacent points is $2A$. Under the additive white Gaussian noise (AWGN) channel, which of the following statement is true?



(a)　　(b)

(a) Both constellations result in the same symbol error rate under the same signal-to-noise ratio.

(b) Constellation (b) results in smaller symbol error rate than constellation (a) under the same signal-to-noise ratio.

(c) Constellation (a) results in smaller symbol error rate than constellation (b) under the same signal-to-noise ratio.

(d) Let E_a be the average transmitted energy of the constellation (a) and E_b be the average transmitted energy of the constellation (b). Then $E_a = E_b$.

(e) None of the above.

8. Consider a four-phase PSK signal represented by the equivalent lowpass signal $u(t) = \sum_{n=-\infty}^{\infty} I_n g(t-nT)$, where I_n takes on one of the four possible values $\sqrt{1/2}(\pm 1 \pm j)$ with equal probability. The sequence of information symbol $\{I_n\}$ is statistically independent. Let $G(f)$ be the Fourier transform of $g(t)$. Determine the power spectral density $S_u(f)$ of $u(t)$.

(a) $S_u(f) = \frac{1}{T}|g(f)|^2$

(b) $S_u(f) = |G(f)|^2$

(c) $S_u(f) = G(f-1/T) + G(f+1/T)$

(d) $S_u(f) = \frac{1}{T}|G(f)|^2$

(e) None of the above.

9. Define $X_+(f)$ as the frequency-domain signal which contains only positive frequency components of $X(f)$. Let $x_+(t)$ be the signal whose Fourier transform is $X_+(f)$. Let $\hat{x}(t)$ be the Hilbert transform of $x(t)$. Which of the following statement is true?

(a) $x_+(t) = \hat{x}(t) + jx(-t)$

(b) $x_+(t) = \frac{1}{2}x(t) + \frac{j}{2}\hat{x}(t)$

(c) $x_+(t) = x(t) + j\hat{x}(t)$

(d) $x_+(t) = \hat{x}(t) + jx(t)$

(e) None of the above.

10. Regarding the functions

$$h_k(t) = \text{sinc}\left(2W\left(t - \frac{k}{2W}\right)\right),$$

which of the following statements is untrue?

(a) $h_k(t)$ and $h_j(t)$ are orthogonal for $k \neq j$.

(b) $\int_{-\infty}^{\infty} (f_k(t))^2 dt = 1/(2W)$.

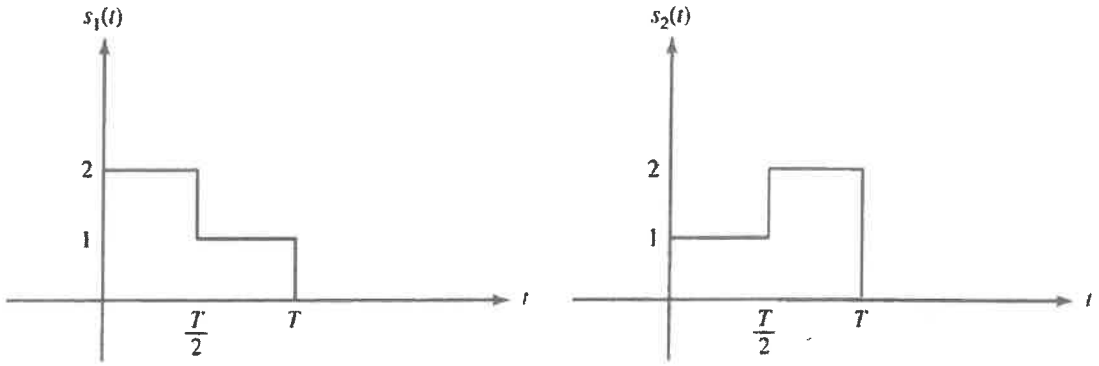
(c) $f_k(t)f_j(t) = 0$ for $k \neq j$.

(d) $\int_{-\infty}^{\infty} f_k(t)f_j(t)dt = 0$ for $k \neq j$.

II. 計算題（共50分）

Give detailed derivations on the following questions. The grade of this portion depends not only on the correct answers but also on the explanations and derivations. Therefore, explain every detail as possible as you can.

1. (25 %) A binary signaling scheme over an AWGN channel with noise power spectral density of $N_0/2$ uses the equiprobable signals $s_1(t)$ and $s_2(t)$ shown below and is operating at a bit rate of R bits/s.



   (a) (5 %) What is E_b/N_0 for this system in terms of N_0 and R?

   (b) (10 %) What is the bit error probability for this system in terms of N_0 and R?

   (c) (10 %) By how many decibels (dB) does this system underperform a binary antipodal signaling system with the same E_b/N_0.

2. (25 %) The discrete sequence

$$r_k = \sqrt{E}c_k + n_k, \qquad \text{for } k = 1, 2, \ldots, n$$

   represents the output sequence of samples from a demodulator, where $c_k = \pm 1$ are elements of one of two possible codewords, $\mathbf{c}_1 = [1, \ldots, 1]$ and $\mathbf{c}_2 = [1, \ldots, 1, -1, \ldots, -1]$. The codeword $\mathbf{c}_1$ is an all-one vector and the codeword $\mathbf{c}_2$ has w elements that are $+1$ and $n - w$ elements that are -1, where w is some positive integer. The noise sequence $\{n_k\}$ is white Gaussian with variance σ^2.

   (a) (10 %) What is the maximum-likelihood detector for the two possible transmitted signals?

   (b) (10 %) Determine the probability of error as a function of the parameters (σ^2, E, w).

   (c) (5 %) What is the value of w that minimizes the probability of error?

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[第 4 節]

科目名稱	線性代數
系所組別	電機工程學系-信號與智慧計算組
	通訊工程學系 通訊甲組 － 通訊丙組

－作答注意事項－

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

國立中正大學 110 學年度碩士班招生考試試題

科目名稱：線性代數　　本科目共 1 頁　第 1 頁

系所組別：電機工程學系-信號與智慧計算組

通訊工程學系-通訊甲組、通訊丙組

1. Consider the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a-1 & b+1 & 0 \\ 0 & b-2 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$. Express your answer with parametric equations in $\mathbf{R}^2$ for the following requirements.

a. (5 pts.) Find all possible a and b for rank 2?

b. (10 pts.) Find all possible a and b for rank 3?

c. (10 pts.) Find all possible a and b for nonsingular matrix.

2. In the linear system

$$-2x_1-2x_3+2x_4 = 4$$
$$x_2-x_3+2x_4 = -2$$
$$x_1+2x_2+x_4 = -3$$
$$x_2-3x_3+7x_4 = -10$$

a. (10 pts.) Find an LU-decomposition of the coefficient matrix.

b. (10 pts.) Find the inverse matrix of the coefficient matrix.

c. (5 pts.) Solve the system with LU-decomposition.

3. Let $\boldsymbol{A}$ be the coefficient matrix of the linear system

$$x-y=1$$
$$-x+2y=0$$
$$2x+y=4$$

a. (5 pts.) Determine the rank of $\boldsymbol{A^TA}$.

b. (10 pts.) Find the eigenvalues and eigenvectors of $\boldsymbol{A^TA}$.

c. (10 pts.) Show the inverse of $\boldsymbol{A^TA}$ with details.

d. (20 pts.) Finish a singular value decomposition of $\boldsymbol{A}$.

Hint: $\boldsymbol{A}$ can be expressed as $\boldsymbol{UBV^T}$ where the column set of $\boldsymbol{U}$ is an orthonormal basis for column space of $\boldsymbol{A}$, $\boldsymbol{B}$ is the diagonal matrix with nonzero diagonal entries which are the square roots (in decreasing order) of eigenvalues of $\boldsymbol{A^TA}$, and $\boldsymbol{V}$ can orthogonally diagonalize $\boldsymbol{A^TA}$.

e. (5 pts.) Find the best approximation for this system.

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試 題

[第 3 節]

科目名稱	機率
系所組別	通訊工程學系-通訊丙組

—作答注意事項—

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6. 試題須隨試卷繳還。

國立中正大學 110 學年度碩士班招生考試試題

科目名稱：機率　　本科目共 1 頁　第 1 頁

系所組別：通訊工程學系-通訊丙組

1) (**15%**) A lock has two buttons: a "0" button and a "1" button. To open a door, you need to push the buttons according to a preset sequence of length at most 4.
   a) (**5%**) How many sequences are there?
   b) (**5%**) Suppose you press an arbitrary 2-bit sequence; what is the probability that the door opens?
   c) (**5%**) If the first try does not succeed in opening the door, you try another number; what is the probability of success?

2) (**10%**) The number N of queries arriving in t seconds at a call center is a Poisson random variable with mean $\alpha = \lambda t$ where λ is the average arrival rate in queries/second. Let Z be the time until the first query arrival.
   a) (**5%**) Is Z a random variable? Justify your answer.
   b) (**5%**) Find the probability of the event $\{Z > t\}$; that is, find $P[Z > t]$.

3) (**20%**) A wire has length which is an exponential random variable with mean π cm. The wire is cut to make rings of diameter 1 cm. Let N be the number of complete rings produced by each length of wire.
   a) (**5%**) Find $P[N = 0]$.
   b) (**5%**) Find $P[N = 1]$.
   c) (**5%**) Find $P[N = n]$.
   d) (**5%**) What type of random variable is N?

4) (**15%**) Let X and Y be independent Rayleigh random variables with parameters $\alpha = 1$ and $\beta = 1$, respectively, such that $Z = Y/X$.
   a) (**10%**) Find the cumulative distribution function of Z.
   b) (**5%**) Find the probability density function (pdf) of Z.

5) (**10%**) Let X be a Gaussian random variable with mean 0 and variance 1.
   a) (**5%**) Find the pdf of $Y = aX + b$, where $a \neq 0$ and b are real numbers.
   b) (**5%**) Find the pdf of $Z = |X|$.

6) (**20%**) The random variable X is uniformly distributed in the interval $[0, a]$, where a is nonnegative. Suppose a is unknown, so we estimate a by the maximum value observed in n independent repetitions of the experiment; that is, we estimate a by $Y = \max\{X_1, X_2, \ldots, X_n\}$.
   a) (**5%**) Find $P[Y \leq y]$.
   b) (**10%**) Find the expected value and variance of Y.
   c) (**5%**) Explain why Y is a good estimate for a when N is large.

7) (**10%**) A fair coin is tossed 100 times. Let N be the number of heads.
   a) (**5%**) Find the expected value and variance of N.
   b) (**5%**) Use the central limit theorem and the Q-function to estimate the probability that N is between 45 and 55.

國立中正大學
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試 題

[第 4 節]

科目名稱	機率
系所組別	通訊工程學系-通訊甲組

—作答注意事項—

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國立中正大學 110 學年度碩士班招生考試試題

科目名稱：機率

系所組別：通訊工程學系-通訊甲組

1) (**10%**) An urn contains three black balls and two white balls. Two balls are selected at random from the urn without replacement and the sequence of colors is noted.
   a) (**5%**) Find the probability of the event that both balls are black.
   b) (**5%**) Find the probability of the event that the second ball is white.

2) (**10%**) Passengers arrive at an airport taxi stand every minute according to a Bernoulli random variable with parameter p. A taxi will not leave until it has two passengers.
   a) (**5%**) Let T be the time when the taxi has two passengers. Is T a random variable? Justify your answer.
   b) (**5%**) Find the probability mass function for the time that the first customer waits.

3) (**10%**) The lifetime X of a machine has a continuous cumulative distribution function (cdf) $F_X(x)$.
   a) (**5%**) Find the conditional cdf given the event $\{X > t\}$.
   b) (**5%**) Find the conditional probability density function (pdf) given the event $\{X > t\}$.

4) (**10%**) Let X be a Gaussian random variable with pdf

$$f_X(x) = \frac{e^{-\frac{1}{2}\left(\frac{1}{4}x^2 - \frac{3}{2}x + \frac{9}{4}\right)}}{\sqrt{2\pi 2}} \quad \text{for} \quad -\infty < x < \infty.$$

   a) (**5%**) Find the expected value of X; that is, find $E[X]$.
   b) (**5%**) Find the variance of X.

5) (**20%**) Let X and Y have joint pdf:

$$f_{X,Y}(x,y) = k(x+y) \quad \text{for} \quad 0 \le x \le 1, 0 \le y \le 1.$$

   a) (**5%**) Find k.
   b) (**5%**) Find the joint cdf of (X,Y).
   c) (**5%**) Find the marginal pdf of Y.
   d) (**5%**) Find $P[X \ge Y]$.

6) (**20%**) Two transmitters send messages through bursts of radio signals to an antenna. During each time slot each transmitter sends a message with probability p. Simultaneous transmissions result in loss of the messages. Let X be the number of time slots until the first message gets through.
   a) (**5%**) Find the conditional expected value of X given that no message gets through in the first time slot.
   b) (**5%**) Show that $E[X|X > 1] = E[X] + \frac{2p(1-p)}{1-2p(1-p)}$.
   c) (**5%**) Find the conditional expected value of X given that a message gets through in the first time slot.
   d) (**5%**) Find $E[X]$ by using the results of parts 6b and 6c.

7) (**20%**) Let X and Y be two random variables. Further, let $W = X - Y$ and $Z = X + Y$.
   a) (**10%**) Find an expression for the joint pdf of W and Z in terms of the joint pdf of X and Y.
   b) (**10%**) Find the joint pdf of W and Z if X and Y are independent exponential random variables with parameter $\lambda = 1$.