# 國立中正大學

# 112 學年度碩士班招生考試

# 試 題

## [第 1 節]

| 科目名稱 | 通訊原理 |
| --- | --- |
| 系所組別 | 通訊工程學系-通訊甲組 |

**—作答注意事項—**

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

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3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1. (20 %) A single sideband (SSB) signal can be expressed as

$$s(t) = m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)$$

where $m(t)$ is the message signal, $\hat{m}(t)$ is the Hilbert transform of $m(t)$, and $f_c$ is the carrier frequency. Let $M(f)$ be the Fourier transform of $m(t)$.

(a) (10%) Determine the Fourier transform of $s(t)$.

(b) (10%) Sketch a block diagram of the demodulator. Show that the message signal can be recovered using the demodulator.

2. (20 %) A DSB-SC signal is represented by

$$s(t) = Am(t)\cos(2\pi f_c t)$$

where $m(t)$ is the message signal and $f_c$ is the carrier frequency. Let $M(f)$ and $S_m(f)$ be respectively the Fourier transform of $m(t)$ and the power spectral density of $m(t)$.

(a) (5 %) Determine the Fourier transform of $s(t)$.

(b) (5 %) Determine the power spectral density of $s(t)$.

(c) (10 %) How to demodulate the DSB-SC signal? Sketch the block diagram of the demodulator.

3. (20 %) The received signal in a binary communication system that employs antipodal signals is

$$r(t) = As(t) + n(t)$$

where $s(t)$ is shown in Figure 1 and $n(t)$ is AWGN with power spectral density of $N_0/2$. The value of $A$ is given by

$$A = \begin{cases} \sqrt{E_s} & \text{if 1 is transmitted} \\ -\sqrt{E_s} & \text{if 0 is transmitted} \end{cases}$$

(a) (5 %) Sketch the impulse response of the filter matched to $s(t)$.

(b) (5 %) Sketch the output of the matched filter to the input $s(t)$.

(c) (5 %) Determine the variance of the noise output of the matched filter at $t = 1$.

(d) (5 %) Assume that $A = \sqrt{E_s}$ and $A = -\sqrt{E_s}$ occur with equal probability. Determine the probability of error as a function of $E_s$ and $N_0$.

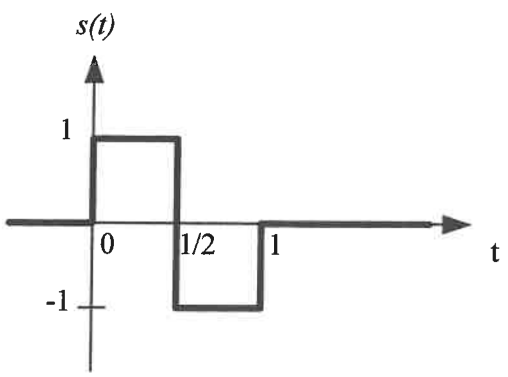


Figure 1

4. (20 %) Three modulation schemes, the binary phase-shift keying (BPSK), the binary frequency-shift keying (BFSK), and the on-off keying (OOK) are considered as candidates for a digital communication system. Let $E_s$ be the average transmit energy per symbol.
   (a) (15 %) Determine the bit error rates for BPSK, BFSK, and OOK over the AWGN channel with power spectral density of $N_0/2$.
   (b) (5 %) Which one should be selected to achieve the lowest bit error rate? Explain why?

5. (20 %) A binary communication system uses two signals $s_1(t)$ and $s_2(t)$, for $0 \le t < T$, to represent equal probable information bit "0" and "1", respectively. The energies for both signals are equal with $E = \int_0^T |s_1(t)|\, dt = \int_0^T |s_2(t)|\, dt$. Let the received signal be $r(t) = s_i(t) + n(t)$, where $n(t)$ is a zero-mean white Gaussian noise with power spectral density of $N_0/2$.
   (a) (10 %) Design an optimal receiver for the system.
   (b) (5 %) Determine the bit error rate for the optimal receiver.
   (c) (5 %) Given $E$ and $N_0$, under what conditions, the receiver has the best performance.

# 國立中正大學
# 112 學年度碩士班招生考試
# 試 題
[第 2 節]

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|---|---|
| 科目名稱 | 線性代數 |
| 系所組別 | 通訊工程學系-通訊甲組 |

—作答注意事項—

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# 國立中正大學 112 學年度碩士班招生考試試題



Let $\mathbf{A} = [\mathbf{A}^{(1)}\ \mathbf{A}^{(2)}\ \mathbf{A}^{(3)}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$, where $\mathbf{A}^{(1)}$, $\mathbf{A}^{(2)}$ and $\mathbf{A}^{(3)}$ are ordered column vectors of **A**,

$\mathbf{B} = [\mathbf{B}^{(1)}\ \mathbf{B}^{(2)}\ \mathbf{B}^{(3)}] = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $\mathbf{B}^{(1)}$, $\mathbf{B}^{(2)}$ and $\mathbf{B}^{(3)}$ are ordered column vectors of **B**, and

$\mathbf{I}_3 = [\mathbf{e}_1\ \mathbf{e}_2\ \mathbf{e}_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ are ordered column vectors of $\mathbf{I}_3$.

1. Show your answers with details
   a. (5 pts.) The sum of all eigenvalues in **A**.
   b. (5 pts.) The geometry multiplicities of **A**.
   c. (5 pts.) The product of all eigenvalues in **B**.
   d. (5 pts.) The inverse matrix of **B** with the augmented matrix $[\mathbf{I}_3|\mathbf{B}]$ and Gauss-Jordan elimination.
   e. (15 pts.) The solution of $\mathbf{A}[x_1\ x_2\ x_3]^T = [1\ 2\ 3]^T$ with Cramer's rule.

2. In $\boldsymbol{R}^3$, find the results with details.
   a. (5 pts.) The transition matrix from the standard basis $\underline{e} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ to the basis $\underline{A} = \{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}\}$.
   b. (5 pts.) The coordinate vector with the basis $\underline{A} = \{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}\}$ corresponding to $(1\ 2\ 3)_{\underline{e}}$ with the standard basis $\underline{e} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$.
   c. (10 pts.) The coordinate vector with the basis $\underline{A} = \{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}\}$ corresponding to $(1\ 2\ 3)_{\underline{B}}$ with the basis $\underline{B} = \{\mathbf{B}^{(1)}, \mathbf{B}^{(2)}, \mathbf{B}^{(3)}\}$.
   d. (10 pts.) The transition matrix from the standard basis $\underline{B} = \{\mathbf{B}^{(1)}, \mathbf{B}^{(2)}, \mathbf{B}^{(3)}\}$ to $\underline{A} = \{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, \mathbf{A}^{(3)}\}$.

3. The inner product is $\langle \mathbf{U},\mathbf{V} \rangle = \mathrm{tr}(\mathbf{U}^T\mathbf{V})$ where **U** and **V** are in the real vector space $\mathbf{M}_{3\times 3}$, and tr(**X**) is the trace of the matrix **X**.
   a. (5 pts.) Find the inner product of the identity matrix ($\mathbf{I}_3$) and **A**.
   b. (10 pts.) Prove or disprove that the additivity axiom holds with this inner product.
   c. (10 pts.) Find the norm-2 length of **B**.
   d. (10 pts.) Show the cosine of the angle between the matrices **A** and **B** with details.

# 國立中正大學
# 112學年度碩士班招生考試
# 試 題
## [第2節]

| 科目名稱 | 機率 |
| --- | --- |
| 系所組別 | 通訊工程學系-通訊甲組 |

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# 國立中正大學 112 學年度碩士班招生考試試題

科目名稱：機率　　　　　　　　　　　　　　　　本科目共 1 頁　第 1 頁
系所組別：通訊工程學系-通訊甲組

1) **(10 points)** Alice has three children. Assume that all eight possible arrangements of boy "$b$" and girl "$g$" in the order of birth, $\{ggg, bgg, gbg, ggb, bbg, bgb, gbb, bbb\}$, are equally probable.
   a) **(5 points)** What is the probability that Alice has two girls and one boy?
   b) **(5 points)** Given the information that at least one of Alice's children is a boy and the younger child is not a girl, what is the probability that Alice has two girls and one boy?

2) **(15 points)** Let $X$ be the number of heads in 10 tosses of a fair coin.
   a) **(5 points)** Find the probability mass function of $X$.
   b) **(5 points)** Find the mean of $X$.
   c) **(5 points)** Find the variance of $X$.

3) **(20 points)** A random variable $X$ has the following probability density function (pdf):

$$f_X(x) = \begin{cases} c(1-x^4), & \text{for } -1 \leq x \leq 1, \\ 0, & \text{elsewhere,} \end{cases}$$

   where $c$ is a constant.
   a) **(5 points)** Find $c$.
   b) **(5 points)** Find the probability that $X > 0$.
   c) **(5 points)** Find the cumulative distribution function (cdf) $F_X(x)$ of $X$.
   d) **(5 points)** Find the mean of $X$.

4) **(15 points)** Consider the following joint pdf of two random variables $X$ and $Y$:

$$f_{X,Y}(x,y) = \begin{cases} x+y, & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

   a) **(5 points)** Find the marginal pdf of $X$.
   b) **(5 points)** Find the probability that "$Y \geq X + 0.5$".
   c) **(5 points)** Are $X$ and $Y$ dependent or independent? Please explain your answer. (0 point if the explanation is incorrect.)

5) **(20 points)** Let $X$ be a normal (Gaussian) random variable with mean 40 and variance 16. Consider $Y = aX + b$, where $a$ and $b$ are two constants to be designed such that $Y$ has zero mean and unit variance.
   a) **(10 points)** Find $a$ and $b$.
   b) **(5 points)** Find the pdf of $Y$.
   c) **(5 points)** Find the characteristic function of $Y$.

6) **(20 points)** Let $X$ be a continuous random variable with cdf $F_X(x)$ and pdf $f_X(x)$. Consider $Y = X^2$.
   a) **(5 points)** The event $\{Y \leq y\}$ is equivalent to what event involving $X$ itself?
   b) **(5 points)** Use part a) to find the cdf of $Y$.
   c) **(5 points)** Use part b) to find the pdf of $Y$.
   d) **(5 points)** If $X$ denotes the amplitude of a radio signal with the following pdf:

$$f_X(x) = \frac{x}{\alpha^2} e^{-x^2/2\alpha^2}, \ x > 0, \ \alpha > 0,$$

   where $\alpha$ is a constant, use part c) to find the pdf of the squared envelope $Y$.